An Interesting Construction

The operation of slide valves for steam locomotives can be analysed through the use of geometrical constructions. Two of these constructions are the Reuleaux diagram and the Zeuner diagram. Full details of both of these constructions can be found in a variety of texts.[1,2] For the purposes of the model engineer, Don Ashton[3] presents a simplified version of the Reuleaux diagram which omits lead and was earlier used by Greenly[4]. A typical example of the simplified diagram is illustrated in Figure 1.

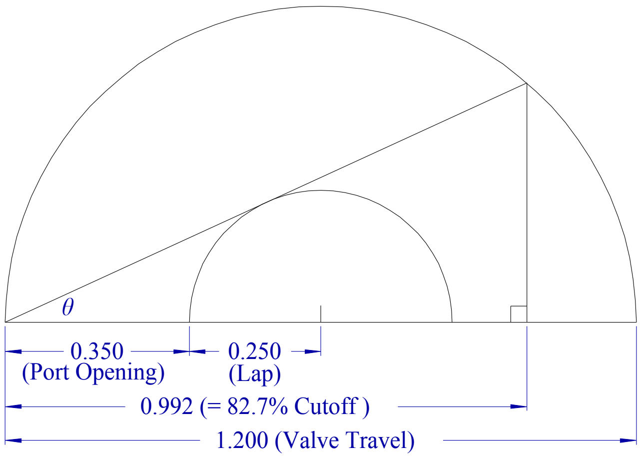

Figure 1. A typical simplified Reuleaux diagram illustrating the relationship between Port Opening, Lap, Valve Travel, and percentage cutoff.

The figure provides a graphical representation of the relationship between valve travel, port opening, lap, and cutoff in full gear. In his book, Mr Ashton includes algebraic formulae for calculations relating to the Reuleaux diagram which permit inclusion of lead. He also describes the use of simulation software for the design of valve gear.

The angle θ is identified as the effective angle of advance. With the inlet port opening "*Line on line*" and the piston simultaneously at the end of its stroke, θ will be the phase difference between valve motion and crank motion. With that relationship established between valve and crank, the inlet port will be closing line on line when the piston has reached the percentage of its travel indicated by Cutoff.

The diagram of Figure 1 only relates to operation of the valve and gives no information regarding the valve gear. There is an underlying assumption that the valve motion will be sinusoidal. Under ordinary conditions, neither the valve motion nor that of the valve gear will be purely sinusoidal.

Figure 1 always has the same general appearance, irrespective of the particular values of its constituent dimensions. The labelled diagram, Figure 2, represents a typical case.

The radius OC and the line BD have been added for explanatory purposes. The radius OC is equal to the lap OF.

Since the line ACD is tangent to the arc at C, the ∠ACO is a right angle. Angle ADB is also a right angle because the arc ADB is a semicircle. The triangles ACO and ADB have a common angle at DAB = θ. They are both right angled triangles. Since the sum of angles in a triangle = 180°, ∠AOC = ∠ABD. The triangles are equiangular and therefore similar. Thus their corresponding sides are in proportion. Since AO is the radius of semicircle ADB, it length is half that of diameter AB. With AO = 1/2AB, it follows that OC = 1/2BD and AC = 1/2AD.

The length of the cutoff line AE is determined by dropping a perpendicular from D to E. Consequently by construction, ∠AED is a right angle. Thus triangle AED is also similar to triangles ACO and ADB.

Although there are apparently five distinct values identified

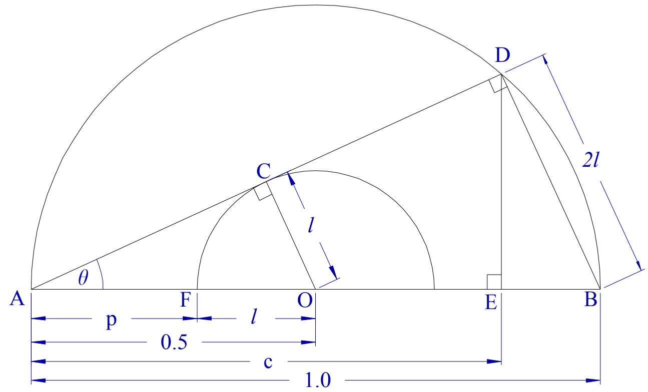

Figure 2. The augmented diagram normalised to a valve travel of 1.0. The construction consists of three similar triangles and two arcs. Its form is completely specified on the basis of any two of the pertinent values.

in Figure 1, the construction requires only two values to be specified in order to define the diagram completely.

For example, assume that the percentage cutoff is given. By reference to Figure 2, the baseline AE can be drawn to represent this percentage. A perpendicular can be drawn at E. The cutoff line AE can be scaled to AB to represent full valve travel. With full travel determined, AB can be bisected to find the centre O. A semicircle of radius OA can now be drawn, intersecting the perpendicular from E at D. This defines the line AD and the angle θ. A perpendicular can be dropped from the line AD at its mid point C, through the centre O, defining the radius OC of the arc for lap. The lap arc is now drawn, intersecting AB at F. The construction is complete. The specification of any one dimension will permit all the other dimensions to be determined by simple proportion.

There are 10 possible pairs of values which might be specified. However, not all of the pairings can be used. As is shown below, the values of lap, percentage cutoff, and the angle θ are implicitly linked.

From a mathematical perspective, it is an interesting exercise to normalise the diagram, making the valve travel equal to unity as shown in Figure 2. The following relationships can then be employed to determine a normalised table of values for all the constituent dimensions.

From the normalised diagram, it can be seen that:

$$p + l = 0.5$$

Considering triangle ACO gives:

$$\sin(\theta) = \frac{l}{0.5} = 2l$$

$$\therefore\ l = \frac{1}{2}\sin(\theta)$$

Alternatively:

$$\theta = \sin^{-1}(2l)$$

The length of AD = 2AC. Consequently, consideration of triangle ADE leads to:

$$c = 2(0.5 \times \cos(\theta)) \times \cos(\theta)$$
$$\therefore c = \cos^2(\theta)$$

Using the trig identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

We have by substitution:

$$c = 1 - \sin^2(\theta)$$

Substituting for $\sin(\theta)$ gives:

$$c = 1 - (2l)^2$$
$$l = \frac{\sqrt{(1-c)}}{2}$$

Alternatively:

Assuming that the designer wishes the Reuleaux diagram to have any significance in respect of the design, the above analysis shows that once the desired full gear cutoff is determined, the value of lap and the angle of advance are also specified. Alternatively, if the angle of advance is known, lap and cutoff are defined. Likewise, any given value of lap corresponds to a specific cutoff and angle of advance. Clearly, it is possible to use incompatible values of lap and angle of advance. This simply results in a design which is not described by the Reuleaux diagram. Nevertheless, the performance of such a design may be regarded as satisfactory.

The normalised Reuleaux diagram may be of interest to model engineers who wish to experiment without persistent recourse to a computer simulation program. Table 1 gives values of each of the variables for a range of cutoffs. Using the table, it is possible to provide answers to a variety of questions.

% Cutoff	Advance Angle	Lap	Port Opening
30	56.8	0.418	0.082
35	53.7	0.403	0.097
40	50.8	0.387	0.113
45	47.9	0.371	0.129
50	45.0	0.354	0.146
55	42.1	0.335	0.165
60	39.2	0.316	0.184
65	36.3	0.296	0.204
70	33.2	0.274	0.226
75	30.0	0.250	0.250
80	26.6	0.224	0.276
85	22.8	0.194	0.306
90	18.4	0.158	0.342
95	12.9	0.112	0.388

Table 1. Valve properties normalised to a full gear valve travel of 1 unit.

For example, assume the measured full gear valve travel was found to be 0.8″, and the angle of advance to be 27°. The nearest table value is 26.6°, which shows the value of cutoff directly as 80%. This, of course, assumes that the lap is correct on the model. The value of lap given in the table is 0.224 normalised to a full gear valve travel of 1. This may represent 1 Angstrom, 1mm, 1″, or 1 light year for that matter. Provided all the dimensions are measured in the same units, the table of values can be employed.

Since the full gear travel in this example is 0.8″, the value of lap found in the table is simply multiplied by 0.8 to arrive at the correct lap corresponding to the actual valve travel. Thus lap should be 0.8 x 0.224 = 0.179″. Similarly, the full gear port opening can be determined by multiplying the table value by 0.8. So the port opening will be 0.8 x 0.276 = 0.221″.

If the valve is measured and its dimensions together with those of the port do not provide the required lap, then the model engineer has the option of modifying the valve or adjusting the angle of advance to achieve compatibility. In the case that neither option is appropriate, the model engineer can use the measured lap and the value given in the table corresponding to the angle of advance, to arrive at a factor representing full gear valve travel. It is then a matter of adjusting the full gear valve travel to achieve that required.

Suppose the lap was found to be 0.150″ instead of the required 0.179″. The factor is obtained by dividing the measured value by the table value. Thus 0.150/0.224 = 0.670″. If the full gear valve travel can be adjusted from its value of 0.8″ down to a value of 0.670″, conditions will be corrected to the Reuleaux design criteria. The resulting full gear port opening will be 0.670 x 0.276 = 0.185″.

Suppose a designer wishes to have a full gear port opening of 5.0mm and cutoff of 70%. From the row in the table corresponding to a cutoff of 70%, the required advance angle is shown to be 33.2°. To determine the full gear valve travel, the appropriate factor is calculated by dividing 5.0mm by the figure in the port opening column. Thus 5.0/0.226 = 22.12 mm. The lap is obtained by multiplying the valve travel by the figure in the lap column. So the required lap is 22.12 x 0.274 = 6.06mm.

Consider a model which has a propensity for wheel spin. One approach to alleviating the problem might be to reduce full gear cutoff. Assume that the model has full gear cutoff of 85% and valve travel of 0.9″. The objective might be to reduce cutoff to 75%. Reference to Table 1 shows that the original advance for 85% cutoff should be 22.8°, while lap will be 0.9 x 0.194 = 0.175″. Port opening will be 0.9 x 0.306 = 0.275″. To achieve 75% cutoff, the angle of advance will need to be increased to 30.0°. If valve travel is to remain unaltered, the lap must be 0.9 x 0.250 = 0.225″. Port opening will be 0.9 x 0.250 = 0.225″. Since the lap would need to be increased, a new slide valve would be required as well as the valve gear being altered to increase the advance. The original valve could be retained by adjusting the valve travel. The existing lap of 0.175″ requires a valve travel of 0.175/0.250 = 0.700″. This in turn will give a port opening of 0.700 x 0.250 = 0.175″.

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